

Progression Towards a Written Method for Division.

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction
- sharing into equal amounts

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e. $15 \div 3$ is not the same as $3 \div 15$
- not associative i.e. $30 \div (5 \div 2)$ is not the same as $(30 \div 5) \div 2$

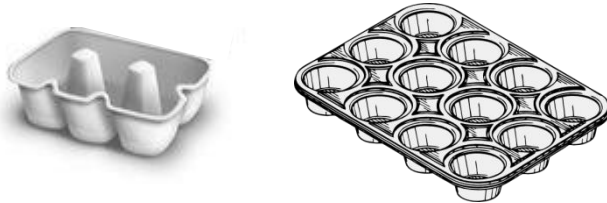
YR

Early Learning Goal:

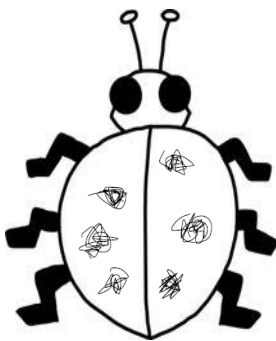
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups.

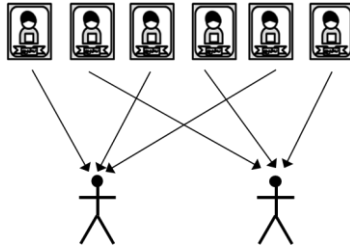


Y1

End of Year Objective:

Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '... left over'.

Y2

End of Year Objective:

Calculate mathematical statements for division within the multiplication tables and write them using the division (\div) and equals ($=$) signs.

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

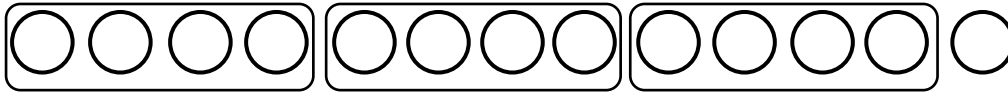
$$12 \div 3 =$$



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

They should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation $13 \div 4$, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still needs to be placed into a box)

End of Year Objective:

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$43 \div 8 =$$



$$43 \div 8 = 5 \text{ remainder } 3$$

In preparation for developing the ‘chunking’ method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

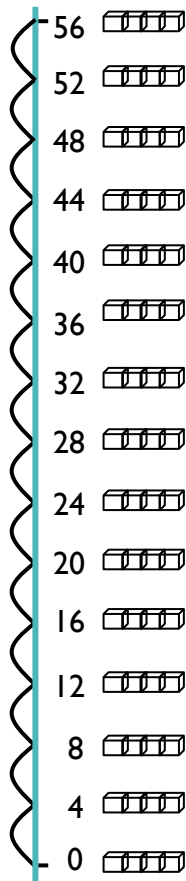
Stage 1 – repeatedly subtracting individual groups of the divisor

Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

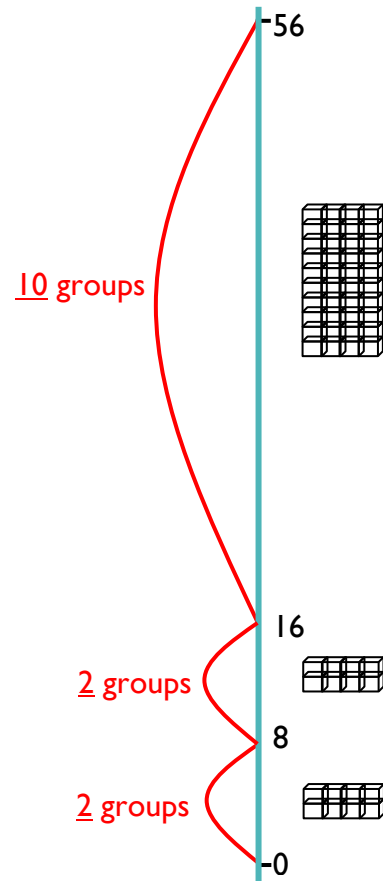
Stage 1

$$56 \div 4 = 14 \text{ (groups of 4)}$$



Stage 2

$$56 \div 4 = 10 \text{ (groups of 4)} + 2 \text{ (groups of 4)} + 2 \text{ (groups of 4)} \\ = 14 \text{ (groups of 4)}$$



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

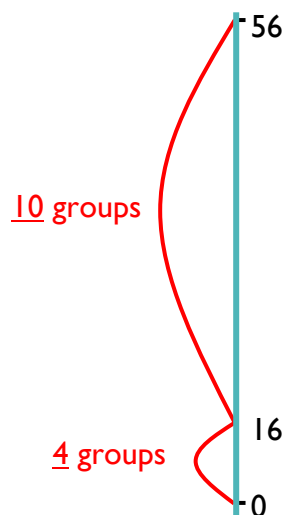
If children are ready, they begin to explore the more formal written methods as described in Y4.

Y4

End of Year Objective:

Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.



$$\begin{array}{r} 14 \\ 4 \overline{) 56} \\ \underline{- 40} (10 \times 4) \\ 16 \\ \underline{- 16} (4 \times 4) \\ 0 \end{array}$$

Answer: 14

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

$$73 \div 3$$

$$\begin{array}{r} 24\text{r}1 \\ 3 \overline{) 73} \\ - 30 \\ \hline 43 \\ - 30 \\ \hline 13 \\ - 6 \\ \hline 7 \\ - 6 \\ \hline \text{r} 1 \end{array}$$

(10 x 3)
(10 x 3)
(2 x 3)
(2 x 3)

Key facts box

1x	3
2x	6
5x	15
10x	30

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

$$196 \div 6$$

$$\begin{array}{r} 32\text{r}4 \\ 6 \overline{) 196} \\ - 120 \\ \hline 76 \\ - 60 \\ \hline 16 \\ - 12 \\ \hline \text{r} 4 \end{array}$$

(20 x 6)
(10 x 6)
(2 x 6)

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

For those who are ready to move on, introduce the more formal method of short division, e.g.

$$98 \div 7$$

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \\ - 7 \\ \hline 28 \\ - 28 \\ \hline \end{array}$$

How many times does 7 go into 9? Place the 1 above and the remainder (2) next to the 8 to make 28. How many times does 7 go into 28? Place the 4 above to give the answer 14.

$$208 \div 5$$

$$\begin{array}{r} 41\text{r}3 \\ 5 \overline{) 208} \\ - 20 \\ \hline 8 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y5

End of Year Objective:

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.

$$523 \div 8$$

$$\begin{array}{r} 65\text{r}3 \\ 8 \overline{) 523} \\ - 320 \quad (40 \times 8) \\ \hline 203 \\ - 160 \quad (20 \times 8) \\ \hline 43 \\ - 40 \quad (5 \times 8) \\ \hline \text{r}3 \end{array}$$

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

$$2458 \div 7$$

$$\begin{array}{r} 351\text{r}1 \\ 7 \overline{) 2458} \\ - 2100 \quad (300 \times 7) \\ \hline 358 \\ - 350 \quad (50 \times 7) \\ \hline 8 \\ - 7 \quad (1 \times 7) \\ \hline \text{r}1 \end{array}$$

Children will be encouraged to use the formal short division method that was introduced in Y4.

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

End of Year Objective:

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal places.

Where appropriate, the children will be encouraged to use the formal short division method that was introduced in Y4 and consolidated in Y5.

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

$$6367 \div 28$$

$$\begin{array}{r}
 227r11 \\
 28 \overline{)6367} \\
 \underline{-5600} \quad (200 \times 28) \\
 767 \\
 \underline{-560} \quad (20 \times 28) \\
 207 \\
 \underline{-140} \quad (5 \times 28) \\
 67 \\
 \underline{-56} \quad (2 \times 28) \\
 r11
 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g. $362 \div 17$

$$362 \div 17$$

$$\begin{array}{r}
 21.29 \\
 17 \overline{)362} \\
 \underline{-340} \quad (20 \times 17) \\
 22 \\
 \underline{-17} \quad (1 \times 17) \\
 5.0 \\
 \underline{-3.4} \quad (0.2 \times 17) \\
 1.60 \\
 \underline{-1.53} \quad (0.09 \times 17) \\
 0.07
 \end{array}$$

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,
e.g. if $2 \times 17 = 34$, I know that $0.2 \times 17 = 3.4$
if $9 \times 17 = 153$, $0.9 \times 17 = 15.3$
so $0.09 \times 17 = 1.53$

An alternative method is the formal written method of long division, sometimes referred to in school as **‘the arrow method’**. This method follows on from the short division method introduced in LKS2, but children calculate their remainders using subtraction at each stage in the division calculation. Children are encouraged to calculate and note their multiples near their division calculation, to support the use of larger numbers.

Long division is set out in the following way:

$$15 \overline{) 3640}$$

$$\begin{array}{r} 2 \\ 15 \overline{) 3640} \\ - 30 \\ \hline 6 \end{array}$$

15 into 3 doesn't go, so look at the next digit.

15 goes into 36 two times, so put a 2 above the 6.
 $15 \times 2 = 30$

Take that 30 away from the 36 to get your remainder.
 $36 - 30 = 6$

$$\begin{array}{r} 24 \\ 15 \overline{) 3640} \\ - 30 \\ \hline 64 \\ - 60 \\ \hline 4 \end{array}$$

Next, carry the 4 down to make 64.

15 goes into 64 four times, so put a 4 above the 4.
 $15 \times 4 = 60$

Take 60 from the 64 to get your remainder.
 $64 - 60 = 4$

$$\begin{array}{r} 242 \\ 15 \overline{) 3640} \\ - 30 \\ \hline 64 \\ - 60 \\ \hline 40 \\ - 30 \\ \hline 10 \end{array}$$

Carry the 0 down to make 40.

15 goes into 40 two times, so put a 2 above the 0.
 $15 \times 2 = 30$

Take 30 from the 40 to get your remainder.
 $40 - 30 = 10$

Multiples of 15

15
30
45
60
75
90
105
120
135
150

For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as $13 \div 4$ to show the remainder initially as a fraction.



Using practical equipment, children can see that for $13 \div 4$, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

$$3574 \div 8$$

$$\begin{array}{r} 8 \overline{) 3574} \\ - 3200 \\ \hline 374 \\ - 320 \\ \hline 54 \\ - 48 \\ \hline r6 \end{array}$$

(400 × 8)
(40 × 8)
(6 × 8)

$$\frac{6}{8} \leftarrow \begin{array}{l} \text{remainder} \\ \text{divisor} \end{array}$$

So $3574 \div 8$ is $446\frac{6}{8}$
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$

Fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

Tenths: $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, $\frac{4}{10} = 0.4$, $\frac{5}{10} = 0.5$, $\frac{6}{10} = 0.6$, $\frac{7}{10} = 0.7$, $\frac{8}{10} = 0.8$, $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, $3574 \div 8$, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75